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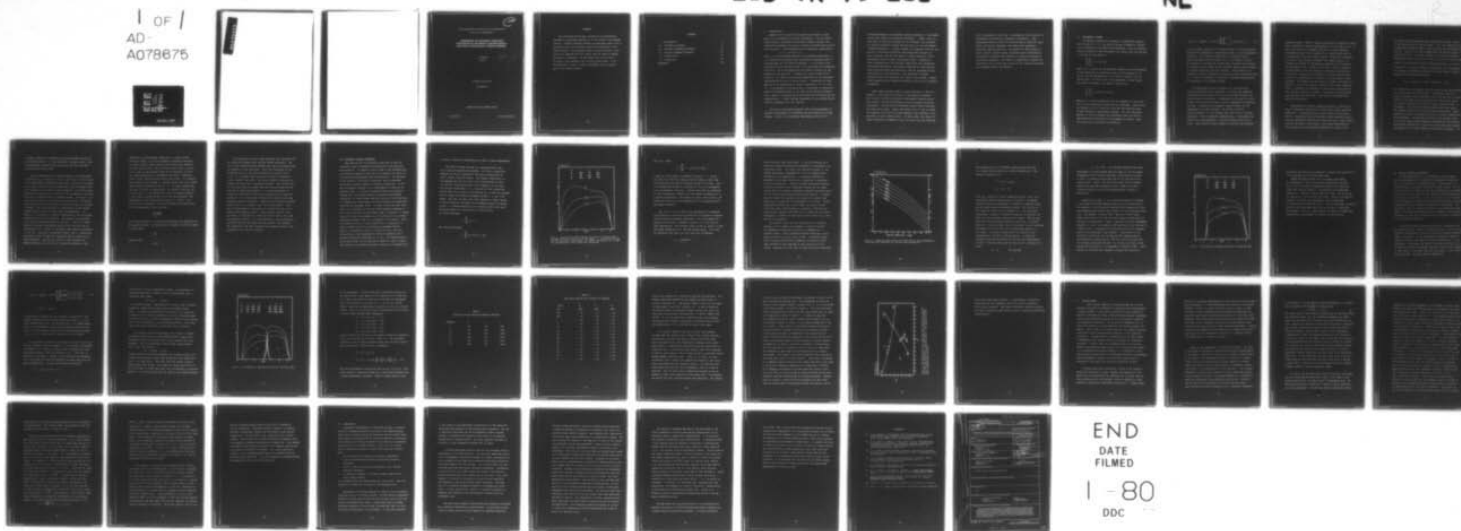
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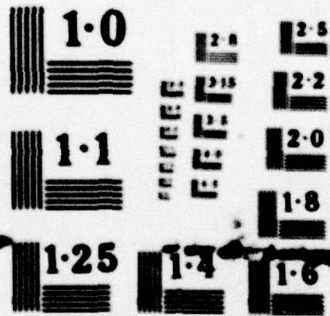
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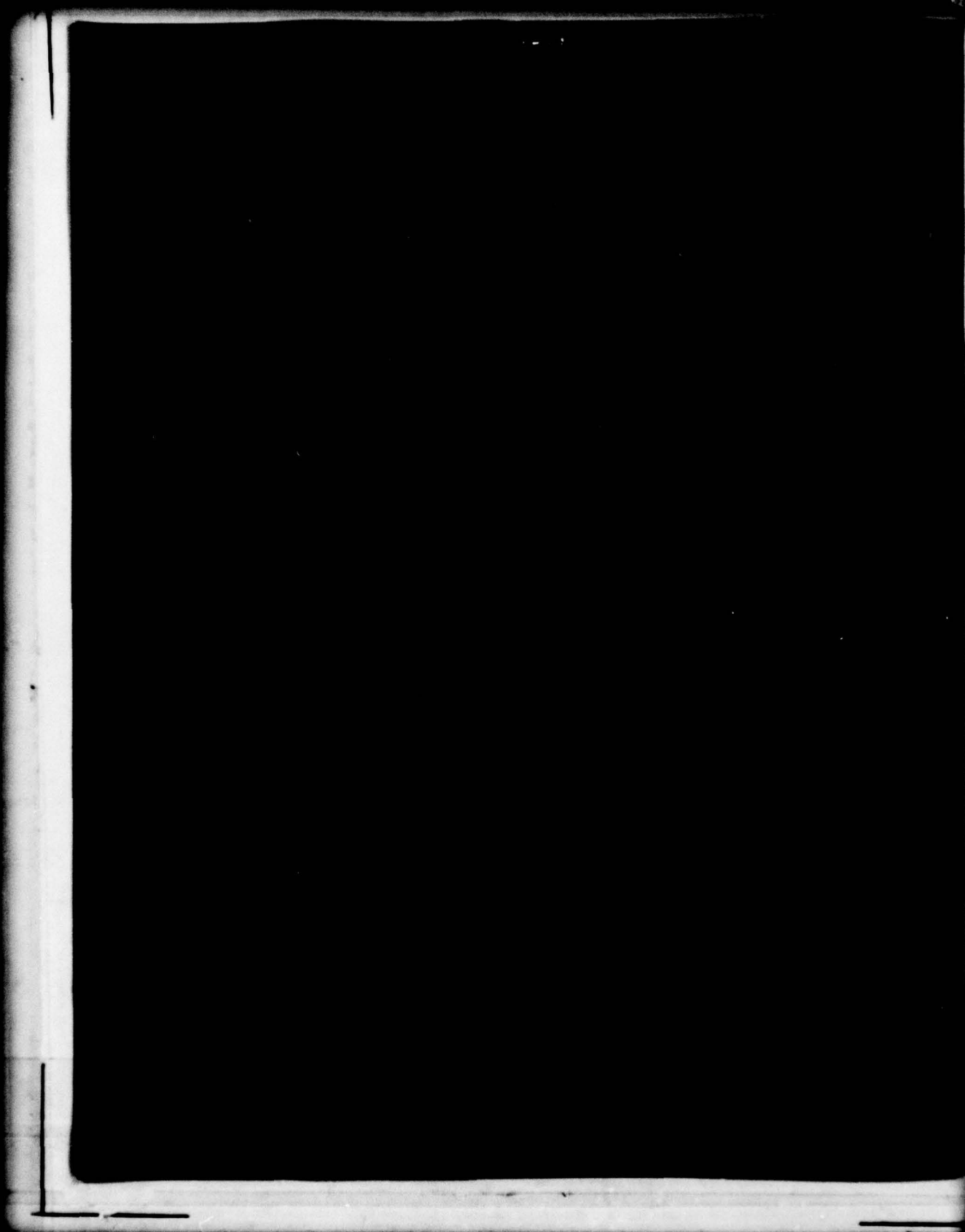
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MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
LINCOLN LABORATORY

PHOTOMETRY OF ARTIFICIAL SATELLITES  
APPLICATION TO THE GROUND ELECTRO-OPTICAL  
DEEP SPACE SURVEILLANCE (GEODSS) PROGRAM

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Group 94

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## ABSTRACT

This note defines the basic concept of an astronomical photometric system and examines it in the context of the GEODSS program. Several different systems are developed, each to fit a particular part of the overall mission for photometry in the GEODSS program. The different kinds of errors arising in each system are examined and their impact upon the goals of GEODSS photometry is assessed. Of particular note is the role of aliasing in the photometry of artificial satellites. It is concluded that if care is taken, photometry can be a useful part of the GEODSS program.



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## I. INTRODUCTION

Amongst the few clues to the nature of an object in deep space is its brightness at optical wavelengths. For the Ground Electro-Optical Deep Space Surveillance (GEODSS) program, this information represents the flux of sunlight reflected from an artificial satellite. The brightness may be represented symbolically:

$$m = a - 2.5 \log f + e$$

where  $m$  is the brightness measured in astronomical magnitudes,  $a$  is a constant which depends upon instrumental parameters such as telescope aperture and spectral bandpass,  $f$  is the photon flux reflected from the satellite, and  $e$  is a correction for atmospheric extinction. The flux depends upon the surface reflectivity and geometry of the satellite. Analysis of a large collection of values of  $f$  in order to derive those properties is only partially understood and will not be taken up here. A discussion of some aspects of the problem may be found in references (1), (2), and (3). The atmospheric extinction may, in principle, be derived by means of measurements made at the time of the satellite observation. Some of the properties of the extinction correction will be mentioned here. A more thorough discussion of this problem may be found in references (4), (5), and (9).

This note is concerned primarily with the establishment of systems of measurement of brightness and the properties of these systems. The use of astronomical magnitudes as the unit of



brightness suggests a relationship between photometry in the GEODSS program and classical astronomical photometry. Indeed, there is a great deal of similarity between the two problems, and astronomical practice provides a useful starting point for the development of GEODSS photometry. It must, however, be remembered that the properties of artificial satellites and astronomical objects are quite different, and that the goals of GEODSS photometry differ considerably from the usual astronomical goals. Examples of these differences are the extremely complicated time variability of satellite brightness, the drastic departure of reflected sunlight from black body characteristics, and the importance of the individuality of each satellite. The techniques of GEODSS photometry should be based upon the needs of the program. Considerable departure from standard astronomical practice can therefore be expected.

Some attempt has been made to retain generality. There is, however, a wide range of variation in astronomical technique due in part to differing goals and instrumental characteristics and in part to honest differences of opinion amongst practicing astronomers. The practical differences between these techniques are often quite small. Rather than spend a lot of time on trade-off analyses, this note will often emphasize one technique to the exclusion of other possibilities. In other cases, the impact of these differences on GEODSS is clear and again only one technique

will be discussed in this note. An example of this situation is provided by the question of photometer calibration. Many astronomers prefer to work with state-of-the-art absolute radiometric calibration standards, and indeed some experiments positively require their use. The benefit to GEODSS observations, however, would be small and seems clearly outweighed by the burden of expensive and difficult to operate and maintain calibration equipment. The reader is encouraged to consult the references cited in this note as well as the research literature for a broader view of the subject.



## II. PHOTOMETRIC SYSTEMS

The physical quantity of interest in a photometric measurement program is  $f(\lambda, t)$ , the flux of photons reflected or emitted to the telescope by the object being measured. Since the measurements are made with real instruments the closest that one can come is a value for the quantity

$$\int_0^{\infty} \int_{t_1}^{t_2} f(\lambda, t) R(\lambda) d\lambda dt$$

where  $R(\lambda)$  is called the instrumental response profile and depends on the properties of the detector used, the throughput of the optics, and the transmission of any filters inserted into the optical path. Actually, since the measurement is made through the earth's atmosphere, the quantity measured is

$$\int_0^{\infty} \int_{t_1}^{t_2} f(\lambda, t) R(\lambda) T(\lambda, \omega) d\lambda dt$$

where  $T(\lambda, \omega)$  is the transmission of the atmosphere in the direction  $\omega$  of the satellite as seen from the telescope. Dealing with the effects of  $T(\lambda, \omega)$  turns out to be the most difficult part of most photometric measurement programs. For the time being, however, it will be assumed that techniques exist for more or less accurately correcting for the atmospheric effects. Thus the magnitude of any object is defined by

$$m_{oj}(t') \equiv (\text{const})_j - 2.5 \log \frac{1}{T} \int_0^\infty \int_{t'-T/2}^{t'+T/2} f(\lambda, t) R_j(\lambda) d\lambda dt \quad (1)$$

Here the naught subscript on  $m$  signifies that it is the magnitude corrected for atmospheric transmission losses - the exo-atmospheric magnitude. The  $j$ -subscript specifies the response profile, usually defined by the  $j$ th filter (e.g., the Johnson V-filter). The constant is an arbitrary normalization constant which removes the effects of such things as the telescope aperture and the detector quantum efficiency from the integral. Traditionally it is adjusted so that all magnitudes coincide for stars of spectral type A0. It will be convenient to consider a different normalization for photometry in the GEODSS program.

The magnitudes defined by equation (1) are called the natural magnitudes of the instrument. Of course two different instruments with different filter sets would have different natural systems of magnitudes. Differences on the order of a few percent will still exist even if all the components are nominally the same. In addition the response changes slightly as the components age or are replaced. The natural system defined by equation (1) thus changes with time even for a single instrument. This is obviously unsatisfactory. Fortunately the solution is straightforward: a reference standard must be defined. There have been many attempts to provide local absolute



standard sources. However, typical accuracies for commercial radiometric standards at optical wavelengths are on the order of 3%. It also turns out to be extremely difficult to avoid systematic differences between the throughputs for a nearby source and for an essentially infinitely distant source. Stars have, therefore, been used as the reference standards even though doing so introduces the problem of simultaneous calibration and correction for atmospheric loss. Because of the need to have a reference standard nearby in the sky, because of potential nonlinearity in the detector response, and because of the variations with wavelength of all properties, it is usual to use more than a single standard. For example, the UBV system of Johnson and Morgan (reference 6) is strictly defined in terms of ten primary standard stars. In practice an extended list including an additional 94 stars serves to define the system. These stars are distributed over the entire sky and cover a wide range of brightness and spectral distribution.

Establishing the primary standards requires a careful program of observation carried out on nights with stable observing conditions. (Astronomers refer to such nights as being photometric.) Enough data must be obtained to allow accurate correction for atmospheric extinction on each night and to allow correction for any aging of the system. This latter requirement in turn requires that the program be spread over a year so that

the group of stars measured, which changes with season, returns to the original and the system closes. It is also important that a physically realistic model be used in making the extinction correction. Once this has been accomplished, the natural system is stabilized and is, in effect, defined by the primary standards. Measurements taken on any given night,  $m'_{01}$  and  $m'_{02}$ , may be transformed to the stabilized natural system magnitudes,  $m_{01}$  and  $m_{02}$ , by means of a relatively simple transformation law. An example of a typical transformation from the so-called nightly values to natural system magnitudes might be:

$$m_{01} = 1.001m'_{01} - 0.023 + 0.002(m'_{01} - m'_{02})$$

The constants in the equation could be obtained from observations of a few standard stars each night. Observations made on a second instrument closely matched to the first may also be transformed to the stabilized natural system of the first by means of a similar equation with typically larger values for the coefficients. The stabilized natural system would then become the standard system for a group of similar instruments. There would be no need to re-establish the set of primary standards for each instrument. Observations of a few of the standard stars of the first system would determine transformation coefficients which would simultaneously correct for small differences between instruments and for small changes in each given instrument. It



is usually possible to transform accurately between similar but not matched systems. For very dissimilar systems, however, such transformations usually become non-linear (and less accurate) and often even multi-valued.

Like any measurement of a physical quantity the measurement of values of  $m_{0j}$  is subject to error. Three sorts of error arise in the measurement of satellite magnitudes: error due to processing of data, error due to the limitations of the instrument, and error inherent in the quantity being measured. Error due to data processing includes error in correction for atmospheric extinction and error in transforming to a standard system. The error in the extinction correction is potentially quite serious. Fortunately this correction need not be made in some of the cases of interest to GEODSS. The error involved in transformation to the GEODSS standard system should be essentially inconsequential. If, however, it is desired to transform magnitudes to the UBV system, substantial error can be expected. The question of processing error will be discussed further in section V. There is a large number of possible instrumental defects which can give rise to substantial error. These are thoroughly explicated in references (7) and (8). By proper instrumental design, care in calibration, and proper choice of reduction technique, most of these can be made negligible. The one most likely to cause problems for GEODSS photometry is error due to inaccurate centering of the

satellite in the photometer field stop. In order to keep this error small, it will be necessary to maintain centering to within a fairly small fraction of the field stop diameter. The last sort of error, the error which gives rise to a *fundamental* limit on the accuracy which can be achieved, is due to the random fluctuation in the photon flux which is detected. This is often called the error due to shot noise. If a measurement consists of an average of  $n$  photon counts, the r.m.s. magnitude of these fluctuations will be  $\sqrt{n}$ . An actual observation consists of a pair of measurements: the object in the sky and the sky alone. If the photon flux due to the object produces  $s$  (for signal) counts and that due to the sky produces  $B$  (for background) the two measurements will be  $B + s$  and  $B$ . The error in their difference is thus  $\sqrt{s + 2B}$  and the relative error of the measurement can be written:

$$\epsilon = \frac{\sqrt{1 + 2B/s}}{\sqrt{s}}$$

For a faint object  $B \gg s$  so that the background flux dominates as the source of error. In this case it is common to define a signal to noise ratio:

$$\rho \equiv \frac{s}{\sqrt{B}}$$

and note that

$$\epsilon \approx \sqrt{2}/\rho$$



The discussion of errors leads naturally into discussion of two trade-off problems which confront GEODSS photometry. The size of the background count depends upon the sky brightness and the diameter of the field stop. Since the fluctuation in the background count is the main contributor to the shot noise error, it is apparent that this error can best be reduced by using a smaller field stop. But using a smaller field stop increases the error due to inaccurate tracking. The optimum size for the field stop will depend upon the sky brightness and the accuracy of centering available. It is not practicable to adjust the field stop to the precise optimum size for each observation. It is clear that a selection of several field stop diameters should be available. Another way of reducing the shot noise error is to increase  $B$  and  $S$  by the same factor. This may be accomplished by increasing the integration time or the width of the response profile. These actions, however, have the effect of degrading the resolution of the data in the time and wavelength domains. The two compromises - keep integration time short and use the widest response profile, and lengthen the integration time while narrowing the response profile - are discussed in the next two sections.

### III. EXTREMELY WIDEBAND PHOTOMETRY

Some applications of photometric data have no need for information about the spectral distribution of the radiation being measured. An example of such a case is the determination of rotational periods of less than a few minutes, for which uncorrected and untransformed data are sufficient. The features of the satellite light curve (signature) used to determine period may be either diffuse or specular reflections. In the case of specular features, it is advantageous to keep the integration time,  $\tau$ , as short as possible because the specular flashes are of very short duration, typically milliseconds. Increasing  $\tau$  to times longer than the duration of the flash does not increase the signal count, but does increase the background count and its attendant noise. In order to keep the shot noise small, it is necessary to use the widest possible bandpass so as to allow detection of the maximum number of photons. A system tailored to these requirements will be referred to as extremely wideband photometry (EWP) and is wideband in two senses. First, the response profile is nearly  $6000\text{\AA}$  wide, as compared to a width of  $800\text{\AA}$ , typical for astronomical wideband filters. Second, an integration time of 1 ms and virtually 100% duty cycle provide a data bandwidth of 500 Hz as compared with a typical astronomical bandwidth of  $\leq 0.1$  Hz. The characteristics of the EWP system will be examined in detail in order to determine how well it performs the task for which it is designed and



in order to assess the applicability of EWP to other measurements.

The detector chosen for EWP is a photomultiplier tube with a gallium arsenide photocathode. No filter is specified although the mirror reflectivity and optics transmission do exert a slight filtering effect. The primary limitations to the bandpass are the red cutoff of the photocathode at about  $.92\mu$  and the strong extinction of the atmosphere in the near ultra-violet shortwards of  $.4\mu$ . Because the total extinction depends upon the air mass at the observation, so does the bandpass. Figure 1 shows the effective bandpass through 1, 2, and 3 air masses. Not only is there less total response at larger values of the air mass but the shape of the bandpass appears to change as well. A response profile may be characterized by many parameters of which the following are of interest here:  
the total throughput

$$E_j \equiv \int_0^{\infty} R_j(\lambda) d\lambda ,$$

the central wavelength

$$\lambda_j \equiv \int_0^{\infty} \lambda R_j(\lambda) d\lambda / E_j , \text{ and}$$

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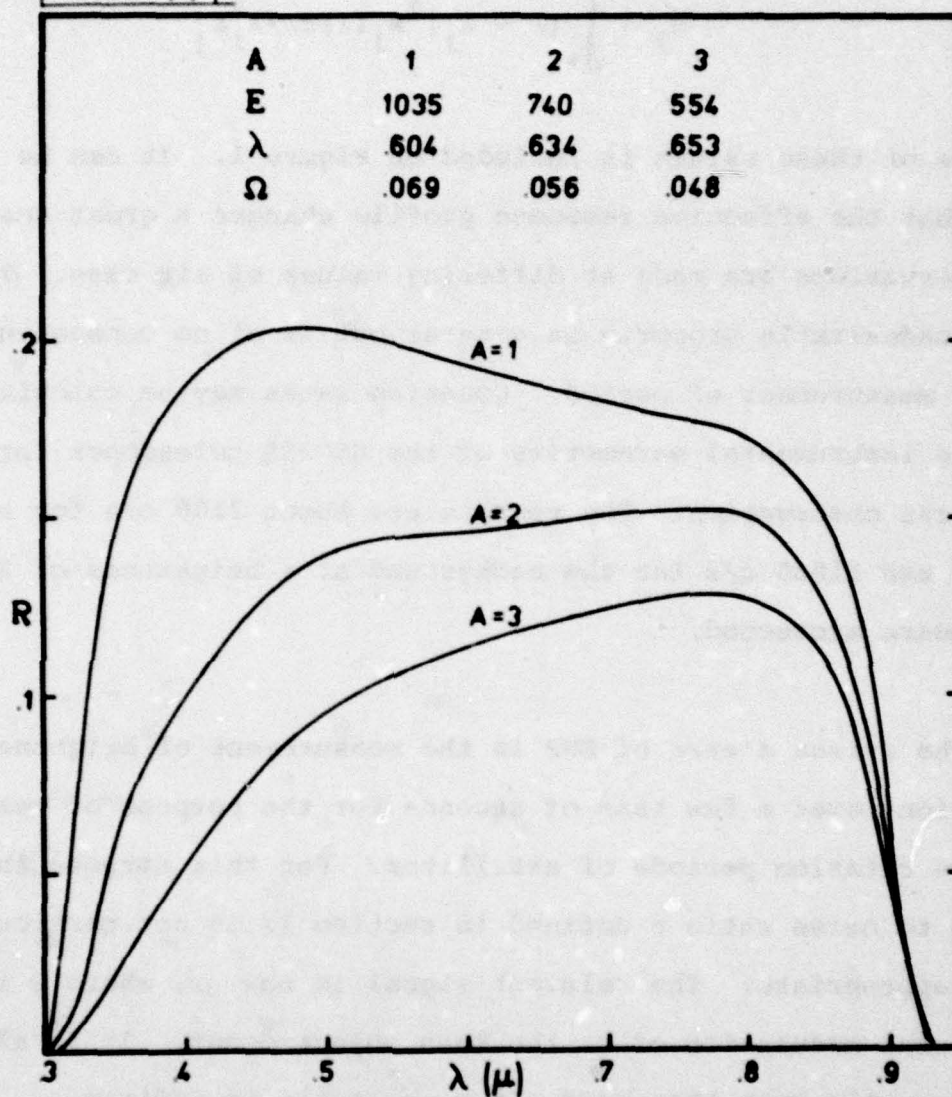


Fig. 1. The effective EWP response profiles for observations through differing values of the air mass,  $A$ . The total throughput (in equivalent Angstroms), the central wavelength (in  $\mu$ ) and the dimensionless r.m.s. width are tabulated.



the r.m.s. width

$$\Omega_j \equiv \int_0^\infty (\lambda - \lambda_j)^2 R_j(\lambda) d\lambda / \lambda_j^2 E_j .$$

A table of these values is included on Figure 1. It can be seen that the effective response profile changes a great deal as observations are made at differing values of air mass. This is an undesirable property in general but is of no consequence to the measurement of period. Counting rates may be calculated for the instrumental parameters of the GEODSS telescopes for a typical observation. The results are about 2100 c/s for a 15<sup>m</sup> object and 11000 c/s for the background at a brightness of 20<sup>m</sup> per square arcsecond.

The *raison d'etre* of EWP is the measurement of brightness variations over a few tens of seconds for the purpose of measuring the rotation periods of satellites. For this purpose the signal to noise ratio  $\rho$  defined in section II is not particularly appropriate. The relevant signal is now  $qS$ , where  $q$  is the fractional modulation of  $S$ , the mean object count. It is also not generally true that  $S \ll B$ , so a new ratio is defined:

$$\rho' \equiv qS / \sqrt{B + S} .$$

Using the count rates given above,  $\rho'$  may be calculated as a function of object brightness and bandwidth of measurement (or integration time). The results are plotted in Figure 2. For diffuse signatures, a bandwidth of 1-3 Hz is adequate and a modulation of 10% appropriate. For specular signatures a bandwidth in the range 100-300 Hz is needed and a modulation of 100% is appropriate. Thus for diffuse signatures attention should be directed to the right-hand ordinate and the upper curves, while for specular signatures attention should be directed to the left-hand ordinate and the lower curves. In either case the signal to noise ratio will need to be greater than about unity in order for the measurement to be useful. If the measurement extends over a large number of cycles, a somewhat lower value could probably be tolerated. Reference to Figure 2 thus indicates that EWP will be useful for its intended purpose. Examples of signatures may be found in Fig. IV-4 and IV-5 of reference 1.

For purposes other than the determination of rotation periods, it will usually be necessary to correct the data for atmospheric extinction and often to transform to a standard system. These processes introduce additional errors which turn out to be quite large in EWP. A detailed discussion of process errors will be deferred to section V; here some estimates of the magnitude of the errors will be obtained. Consider first the case in which extinction correction



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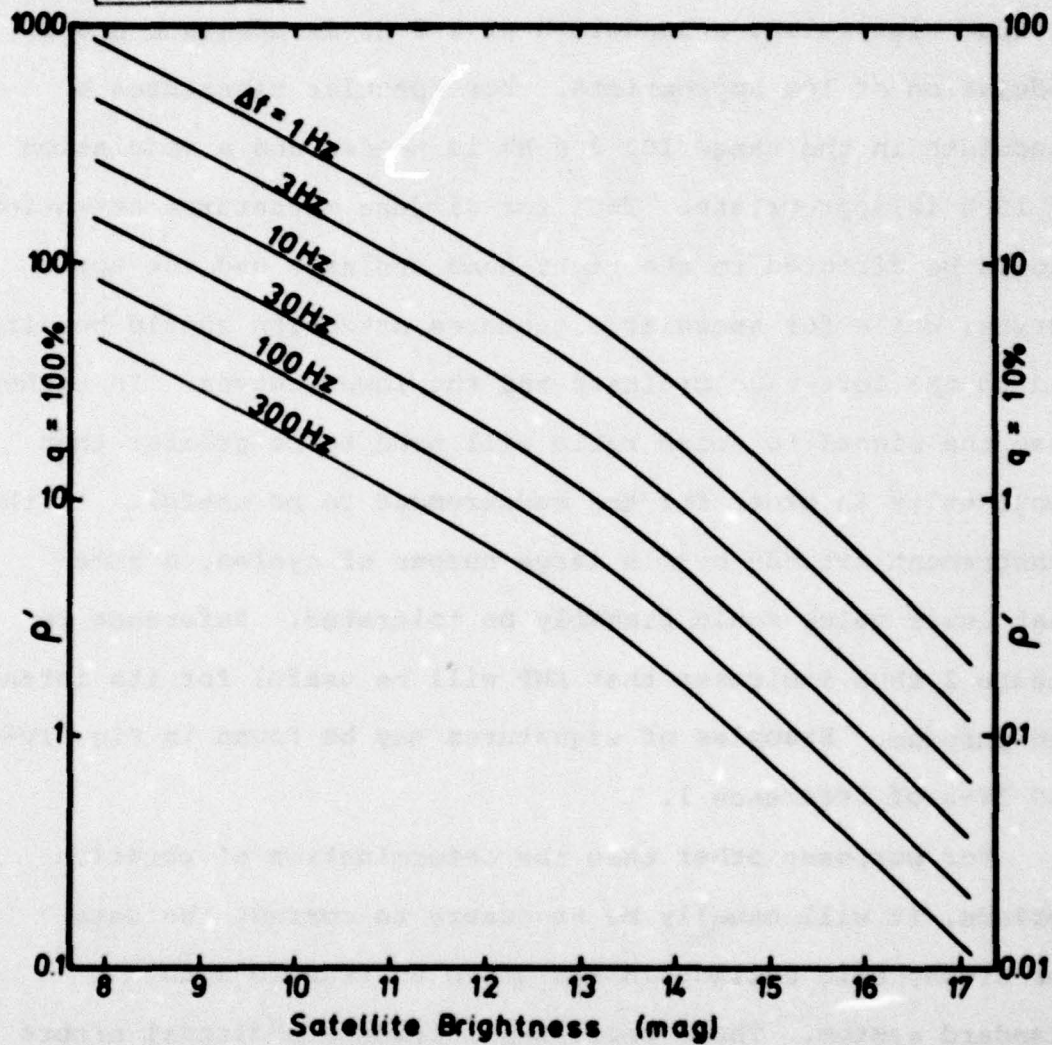


Fig. 2. Signal to noise ratio as a function of (exo-atmospheric) magnitude and bandwidth for 10% and 100% modulation.

and transformation to the GEODSS standard system are made in a single step by measuring a nearby standard star. The reduction equations are:

$$k = (m'_c - m_{co}) / x_c$$

$$m_{po} = m'_p - kx_p$$

where the c-subscript denotes comparison object (star), the p-subscript denotes program object (satellite), the naught subscript denotes exo-atmospheric (standard or catalog) value and the prime denotes the measured values. The validity of this technique rests on the constancy of  $k$ , the extinction coefficient. However, calculations reported in reference (4) show that the extinction coefficient for EWP is .348 for sunlight and .295 for sunlight reflected from gold. No better value for  $k$  than the solar value is available, so this technique introduces an error of  $0^m.13$  into a measurement made at 2.5 air masses. A similar problem arises in connection with transformation from EWP to UBV photometry. An empirical transformation has been derived at the ETS from measurements of stars. Combining this with the data given in reference (4) gives:

$$m_v = m_e \quad \text{for sunlight}$$



$$m_v = m_e + .10 \quad \text{for sunlight reflected from gold.}$$

Measurement of a gold covered satellite made at 2.5 air masses, transformed to the V-filter magnitude, would thus be in error by  $-.23^m$ . It should be noted that this is not the sum of two random errors. Rather, the extinction error and the transformation error are both systematic errors. These two errors correlate perfectly and thus add algebraically instead of vectorially.

Referring to Figure 1, it is clear that most of the change in the shape of the EWP response profile is at the blue end of the spectrum. It would thus be expected that elimination of the blue response would improve the accuracy of the extinction correction. For example, a yellow filter (such as a Schott Glass GG435) will block out the blue end while passing essentially all the rest of the spectrum. Figure 3 shows the effective bandpasses through 1, 2, and 3 air masses for EWP modified by the addition of a GG435 filter. As expected the characteristics of the response profile are considerably less variable. Calculations of the extinction carried out for the modified EWP response profile give extinction coefficients of .283 and .264 for sunlight and for sunlight reflected from gold. As a result the error in the extinction correction at 2.5 airmasses is reduced to  $0.05^m$ . Introduction of the GG435 was intended to improve the extinction

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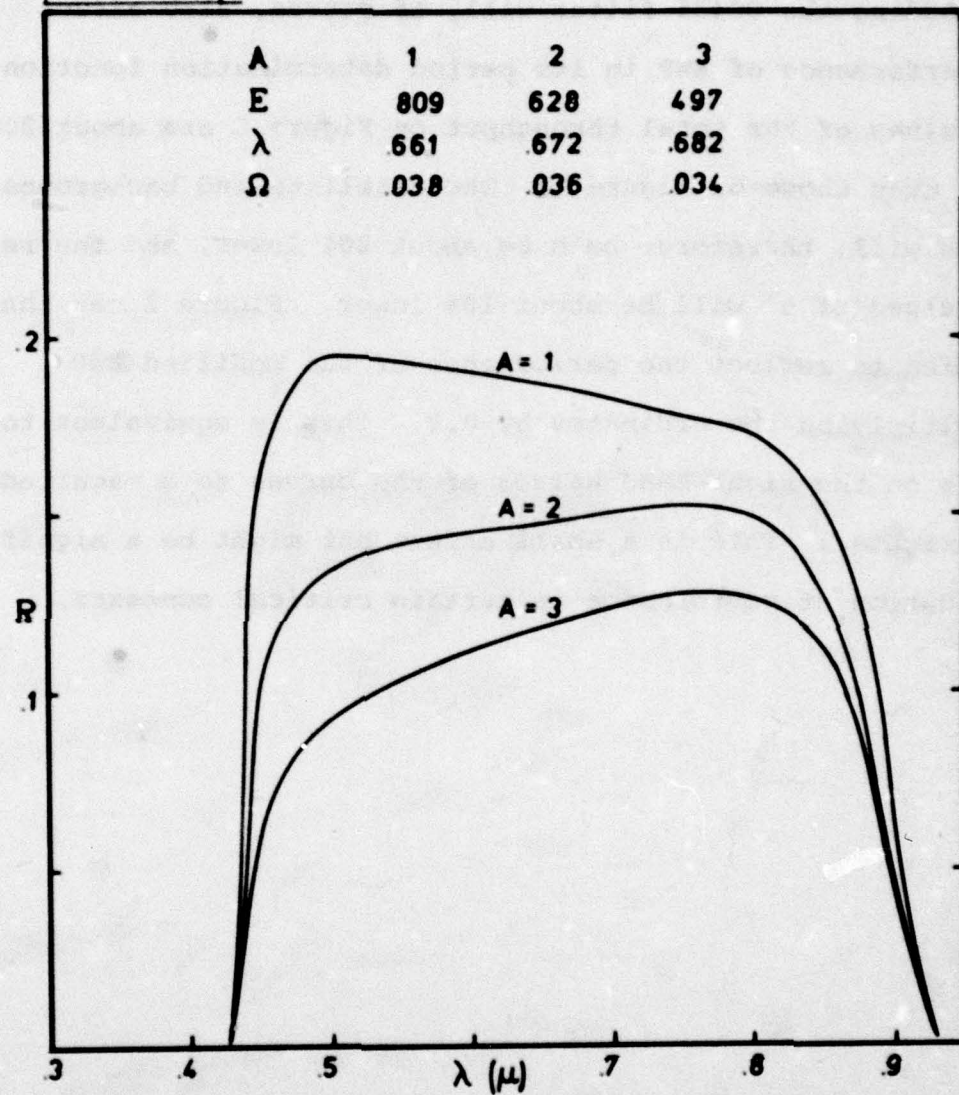


Fig. 3. The effective response profiles for modified EWP.



correction and would not be expected to improve the properties of the transformation to V-filter magnitudes.

Adding the GG435 filter will, of course, also affect the performance of EWP in its period determination function. The values of the total throughput on Figure 3 are about 20% lower than those on figure 1. The satellite and background counts will, therefore, both be about 20% lower, and the resulting values of  $\rho'$  will be about 10% lower. Figure 2 can thus be modified to reflect the performance of the modified EWP by multiplying the ordinates by 0.9. This is equivalent to moving points on the right-hand halves of the curves to a magnitude  $0^m.1$  brighter. This is a small effect but might be a significant degradation of performance in certain critical contexts.

#### IV. MULTIPLE BANDPASS PHOTOMETRY

In most applications, it is necessary to correct photometric measurements for the effects of atmospheric extinction and to put the data on a standard system. Occasionally it is also desirable to transform the data to another system. The results of section III make it clear that information on the spectral distribution will be needed if the correction and transformation are to be done accurately. It may, of course, be that the spectral distribution is itself the interesting data since it provides information about the surface reflectivity of satellites. The possibility that the spectral distribution (alternatively, spectrum) is the desired information will be pursued in this section. Discussion of the role of spectral data in extinction correction will be deferred to section V.

In order to measure spectral distribution, magnitudes must be measured through a number of different filters. The resulting system of measurement will be called multiple bandpass photometry (MBP) in this note. In the astronomical literature, such systems are called by such names as 4-color photometry or  $u, v, b, y$ -photometry. The greater the number of distinct profiles,  $R_j(\lambda)$ , utilized the better the spectral resolution which can be obtained. If just two  $R_j(\lambda)$  are used, then only one number, a color index, can be derived. The color index is defined by:



$$c_0(t') \equiv (\text{const}) - 2.5 \log \frac{\int_0^\infty \int_{t'-\tau/2}^{t'+\tau/2} f(\lambda, t) R_1(\lambda) d\lambda dt}{\int_0^\infty \int_{t'-\tau/2}^{t'+\tau/2} f(\lambda, t) R_2(\lambda) d\lambda dt} \quad (2)$$

$$= m_{01}(t') - m_{02}(t')$$

As mentioned in section II, the constants in equation (1) are normally adjusted so that all magnitudes are equal - i.e., all color indices equal zero - for A0-type stars. For GEODSS photometry, it is convenient to have this coincidence occur for sunlight. If finer spectral resolution is needed several profiles -  $R_1, R_2, \dots, R_n$  - may be used to provide  $n-1$  color indices.

It is possible to optimize the spacing and shape of the  $R_j(\lambda)$  from an information theory standpoint. This problem is discussed in reference (9). There are usually other considerations, however, which severely limit the choice of  $R_j(\lambda)$ . Note that in equation (2) the same  $t'$  appears in both numerator and denominator. This means that strictly speaking the measurements must be made simultaneously. In most astronomical applications

$$(t_2 - t_1) \dot{f}(\lambda, t) / f(\lambda, t) \ll 1$$

so that this is not a significant problem. For photometry of artificial satellites, however, this no longer holds, and a satellite color index

$$CI = m_{01}(t_1) - m_{02}(t_2)$$

is a worthless number. Measurements of a color index in GEODSS photometry should thus be made using a two channel photometer. In order to minimize shot noise error, it is important to make the most efficient possible use of the incoming photons. This implies use of a dichroic beamsplitter which in turn implies more or less rectangular, non-overlapping profiles. This seemingly minor point will play an important role in section V.

One possible system for measuring color indices in GEODSS photometry would consist of two passbands which roughly split the gallium arsenide response into a green response and a red response. The two measurements could be made simultaneously and a color index formed:

$$C0 = m(\text{green}) - m(\text{red})$$

Figure 4 shows the effective passbands for response profiles defined by the reflected (green) and transmitted (red) beams split with a .65 $\mu$  edge filter. The green beam also passes through a Schott Glass GG375 filter. The reason for using this filter is not, as in EWP, to lessen the error due to differences in spectrum. Rather it is to avoid having the blue edge of the bandpass defined



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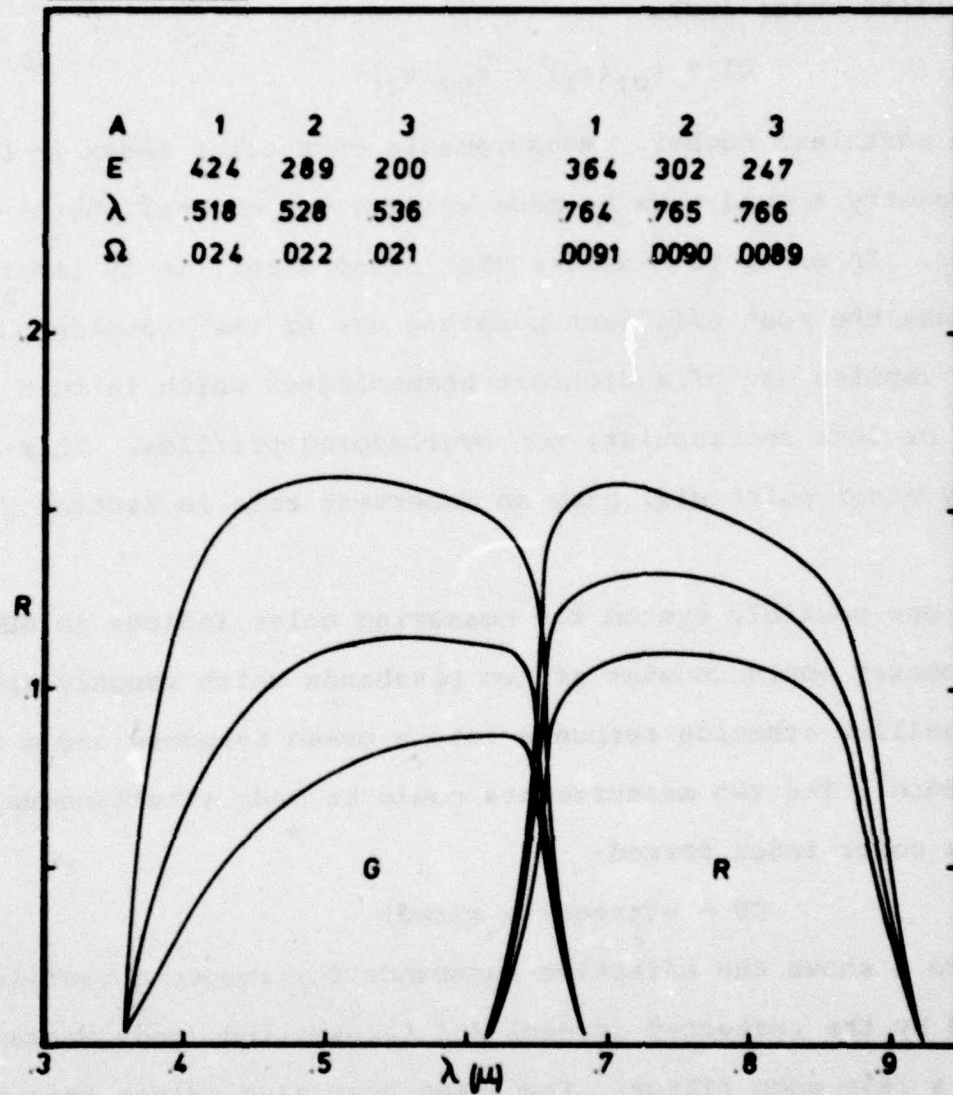


Fig. 4. The effective response profiles for two-color MBP.

by the atmosphere. The blue edge here is defined primarily by the GG375 filter, thus improving the properties of the MBP system. When finer spectral resolution is desired the bandpasses can be split again to give four  $R_j(\lambda)$ : G1, G2, R1, and R2. Table I shows the exo-atmospheric properties of the six profiles of MBP. There is a variety of color indices which may be formed from the counts through these bandpasses:

$$C0 \equiv -2.5 \log G/R + .316$$

$$C1 \equiv -2.5 \log G1/R1 + .243$$

$$C2 \equiv -2.5 \log G2/R2 + .387$$

$$CG \equiv -2.5 \log G1/G2 - .030$$

$$CR \equiv -2.5 \log R1/R2 + .114$$

where G stands for the value of the double integral of equations (1) or (2) with the  $R_G(\lambda)$  profile, and so on. The constants listed here are the values needed to make all the color indices zero for sunlight. These are not independent:

$$C1 = CG - CR + C2$$

$$C0 = C2 - 2.5 \log \frac{10^{(-.012 - .4CG)} + 1}{10^{(.046 - .4CR)} + 1} - .071$$

The three independent indices used here are C0, C1 and C2. This choice allows a convenient design for a dual-beam photometer and a simple measurement procedure. Table II shows values of the



TABLE I  
PROPERTIES OF THE MULTIPLE BANDPASS PROFILES

Bandpass	E	$\lambda_c$	$\Omega$
G	636	.506	.026
G1	303	.444	.0096
G2	231	.587	.0049
R	444	.763	.0093
R1	188	.701	.0027
R2	189	.825	.0026

TABLE II  
MBP COLOR INDICES FOR A VARIETY OF OBJECTS

Object	C0	C1	C2
Sun	0 <sup>m</sup> .00	0 <sup>m</sup> .00	0 <sup>m</sup> .00
WP1	- .04	.00	- .08
WP2	.00	+ .05	- .05
C	- .28	- .85	+ .50
H	- .26	- .50	+ .03
Al	- .01	+ .01	- .03
Au	+ .45	+ .84	+ .17
44	+ .53	+ .65	+ .44
58	+ .02	+ .00	+ .04
69	- .24	- .33	- .15
X1	+ .12	- .05	+ .29
X2	+ .05	- .19	+ .32
X3	+ .01	- .28	+ .34
X4	- .02	- .35	+ .36
Y	+ .27	+ .32	+ .23



three color indices for a variety of spectral distributions. The objects labeled WP1 and WP2 are two samples of white paint as given in reference (2). The objects labeled C and H are two types of solar panels, also from reference (2). Al and Au stand for the reflectivities of aluminum and gold, and 44, 58, and 69 for black body distributions calculated for  $T = 4400^{\circ}\text{K}$ ,  $5800^{\circ}\text{K}$ , and  $6900^{\circ}\text{K}$ . X1, X2, X3, and X4 are composite object distributions. Calculations were based on reflection of sunlight from  $1\text{m}^2$  gold and respectively 5, 7, 9, and  $11\text{m}^2$  of type C solar panel.

It is readily apparent that the C0 color index depends strongly upon the material which reflects the incident sunlight. As expected, such white surfaces as aluminum and white paint leave the solar color index essentially unchanged, while solar panels shift it significantly toward the blue and gold shifts it toward the red. If satellites were covered either with aluminum, or gold or solar panels, then C0 would provide an excellent discriminant amongst satellite types. There is, of course, no such limitation, and in reality the problem is confused by such situations as presented by the composite objects. By combining blue solar panels and red gold, any intermediate value of C0 may be obtained. Thus an object with a composition between X3 and X4 produces a value of C0 identical to the solar value. Fortunately, the use of two color indices resolves this ambiguity. The indices

C1 and C2 will be used here although in principle C0 and C1 or C0 and C2 should work equally well. The information in these two indices is most easily viewed in a plot of C2 vs. C1 - a so-called color-color diagram - as given in figure 5. Here the black body values are plotted as a curve with the values marked for the specific temperatures listed in Table II. As would be expected, the composites X1-X4 lie along a smooth curve between C and Au. A noteworthy feature of figure 5 is the sparse population of the C1-C2 plane. A wide variety of spectral distributions could be plotted on Figure 5 without leading to any crowding of points. In particular the point corresponding to the composite with solar C0 lies far from the solar point (the origin) on the C1-C2 plane, and therefore these two distributions are easily distinguishable on the basis of two-color photometry. Unfortunately, the fact that there is plenty of space on the C1-C2 plane does not necessarily imply that each distribution can be uniquely represented. For example, the smooth composite curve intersects the black body curve at a point labeled Y. This point corresponds to a composite made of areas of solar panels and gold in the ratio 1.9:1 or to a black body radiation at a temperature of 5030°K. As it happens, both spectral distributions also yield a value for C0 of 0.<sup>m</sup>27, so as far as MBP is concerned these two entirely different spectral distributions are indistinguishable. However, this is actually a rather artificial problem since MBP could never be called upon to distinguish between two spectra, one of



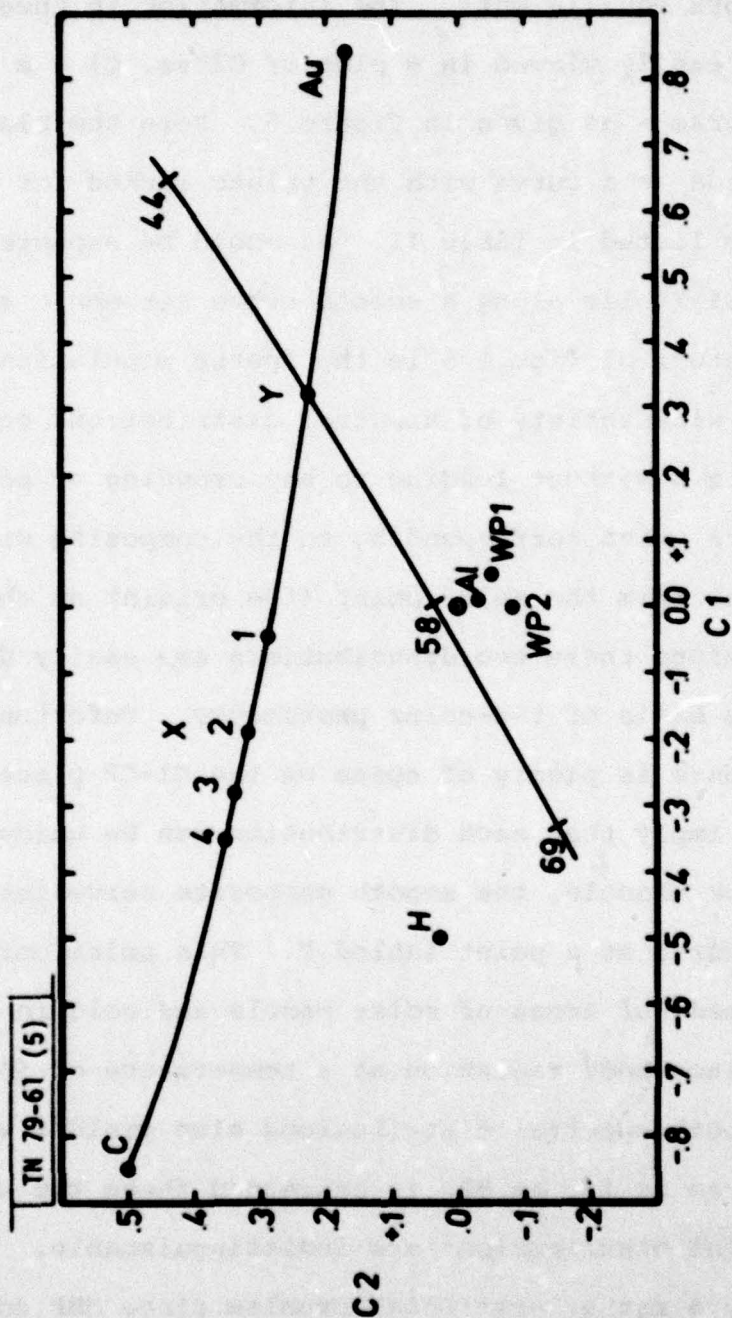


Fig. 5. Color-color plot for sunlight reflected from a variety of satellite materials. The curve with 69, 58 and 44 marked on it is the locus of black body values. The symbol C is the point representing sunlight.

which was a black body spectrum. In particular, it should be noted that the C-Au composite curve and the Al-Au composite curve do not intersect. MBP should, therefore, provide an effective set of parameters upon which to base a satellite reflectivity classification.



## V. PROCESS ERROR

The original reason for considering MBP was to allow accurate correction for atmospheric extinction and transformation to other systems. It was seen in section III that treating all spectral distributions alike in EWP precluded accurate transformation to other systems and gave rise to an inaccurate correction for atmospheric extinction. Use of several bandpasses will improve the situation, but uncertainties from several sources will remain. One of these is the great width of the response profiles, which gives rise to uncertainties primarily in the extinction correction. A second source of uncertainty is the small number of bandpasses used, sharply limiting the amount of information available. Finally, there is a problem having to do with the shapes of the profiles which gives rise to uncertainties primarily in transformation between systems. It is possible to view the extinction correction as a transformation between ground-based and exo-atmospheric systems, and thus this final source of uncertainty will affect the extinction correction as well.

In making extinction corrections, a model of the physical process of extinction is used. Because the parameters of the model must be obtained from the observations, the model must be fairly simple and of low polynomial order as opposed to a more realistic, complicated, exponential description. A simple model

would be an excellent approximation if the extinction were not wavelength dependent or if the measurements were made in a very narrow bandpass. Neither of these conditions is true in EWP or in MBP. One result is that parameters which describe the atmosphere will actually depend upon the spectral distribution of the object being measured. Thus a blue spectral distribution will suffer more total extinction in the atmosphere than will a red distribution, even though the flux may be equal at some specified wavelength. A second result is that parameters which describe the response profile depend upon the total atmospheric extinction. An example of this was already seen in the data of Figure 1.

There are two approaches to solving this problem. The first is to narrow the bandpass to make physical reality conform better to the model, as was done in section III by adding a GG435 filter to the system. The spectacular success, a factor of 2.8 reduction in the error resulting from a slight reduction in width, comes about because most of the variation in atmospheric extinction occurs in the excluded part of the spectrum. A far greater reduction in width would be needed to work an additional factor of two reduction in error. The second approach is to attempt to model the variations in the extinction coefficient. Values of the extinction were calculated for sunlight and for black body radiation by direct integration for a "standard" atmosphere



in reference (4) On the basis of these calculations, it is possible to represent the atmospheric extinction by:

$$k = 0.^m348 - .14 C_0 \quad (3)$$

The actual value of the extinction coefficient for sunlight reflected from gold (again, for the "standard" atmosphere) is  $0.^m295$ .

Using the solar value of  $k$  results in an error of  $0.^m053$  per air-mass. Equation (3) predicts a value of the extinction coefficient for sunlight reflected from gold of  $0.^m285$  yielding an error of  $0.^m010$  per airmass - better than a factor of five improvement.

A combination of these two approaches should be very effective. Calculation similar to that above, carried out for the extinction in the modified EWP system yields an error of  $0.^m002$  per airmass. The magnitude of a not too complicated spectrum measured through the G2-filter and corrected via a model utilizing  $C_1$  could be expected to contain essentially no color error in the extinction correction. On the other hand, the shot noise error in the G2-filter would be more than twice the shot noise error in EWP for a given object, limiting use of the narrower filters to somewhat brighter objects or longer integration times.

Of course, the EWP magnitude cannot be corrected using equation (3) unless MBP has been done as well. This leads to the second problem area: the small number of bandpasses used. The spectral distribution of black body radiation can be specified by a single parameter. The physical quantity ordinarily used is

temperature, but many parameters,  $C_0$  for example, could be used. Earlier, the value  $C_0 = 0.27$  was seen to correspond to the black body temperature  $5030^{\circ}\text{K}$ . There is no other *black body* distribution which has the same value of  $C_0$ . A two bandpass system would thus be sufficient for measurement of black body radiation. Astronomers find that normal stars may be adequately classified with three parameters (physically: temperature, surface gravity, and metal abundance), and thus they use a system (usually a combination of systems) with four or more bandpasses. Unfortunately, the spectra of satellites are very complicated. No classification scheme yet exists, but based on the list of possibly important physical parameters one would expect perhaps several dozen photometric parameters to be necessary in order to study satellites. Fortunately, a "study" isn't necessarily what is called for, and it is possible that useful classification could be done with a small fraction of that number of bandpasses. This is similar to the astronomical case where the single photometric parameter, (B-V), can be very useful in classifying stars. Nonetheless it seems likely that the four bandpasses available in MBP are too few to provide an accurate description of satellite spectra. This has two effects: first there are classification ambiguities such as exist for the composite spectra in table II, and second the information needed to make accurate corrections or transformations may just not be available. This latter is one of two reasons for the qualification above that the accurate



correction for a G2-filter magnitude applies only to sufficiently simple spectra. The second reason, the possibility that the information may be false, is taken up next.

Earlier it was mentioned that the rectangular bandpasses of MBP could give rise to some uncertainties. This occurs because the steep sides of the profiles make the magnitudes sensitive to much of the fine detail in the spectra, but the small number of bandpasses does not provide enough space to report all the information contained in the detail. The few parameters produced must then represent many possible distributions for each set of values. For example  $C_0 = 0^m.00$  represents both sunlight and the composite X3, but the actual distributions are quite different, as can be seen from the values of  $C_1$  and  $C_2$ . Inserting the value  $C_0 = 0^m.00$  into equation (3) produces the result that the extinction coefficient for X3 is  $0^m.348$  - the same as for sunlight. Calculation by direct integration, however, yields the value  $0^m.387$ . The problem is that the information contained in the value of  $C_0$ , which might be stated: 'X3 is just like sunlight', is false. From the standpoint of extinction, the correct value for X3 would be  $C_0 = -0^m.28$ . More generally, color indices represent the slopes of the spectral distributions at a mean wavelength. For closely spaced narrow response profiles, in fact

$$C_0 \sim \left. \frac{d \ln f(\lambda)}{d \ln \lambda} \right|_{\lambda = (\lambda_1 + \lambda_2)/2}$$

where  $\lambda_1$ , and  $\lambda_2$  are the central wavelengths of  $R_1$  and  $R_2$  (see equation 2). Values of the derivative appear in the model, and the success of the model then depends in part on the accuracy of the values used. Returning to the case of the G2-filter magnitude: if the spectrum of the object being measured is smooth, then C1 will accurately represent the slope of the spectrum at the central wavelength of the G2-filter ( $.57\mu$ ), and an accurate correction will result. If there is a strong, narrow extra reflection in the G1 bandpass (i.e., too complicated a spectrum) then C1 will not accurately represent the slope at  $.57\mu$ , and an inaccurate correction will result.

It is possible to describe this phenomenon more formally. The steep sides of the bandpass mean that the Fourier transforms are very wide. This in turn means that they transmit information which has a high "spatial" frequency - i.e., variations in  $f(\lambda)$  which have many maxima and minima in the range of  $\lambda$  covered. In terms of this high frequency data the small number of bandpasses leads to gross undersampling, so that the information at high frequencies is aliased into the parameters which represent the lower frequency data. These parameters, the three color indices of MBP, are thus inaccurate in that they contain unknown contributions from data other than the low frequency information they are supposed to represent. For smooth spectra, this is not



much of a problem because there is very little information at high frequencies. Satellite spectra, however, are complex and have a significant fraction of their data at high spatial frequencies. For satellites, then, aliasing can be a severe problem. The solution is to narrow the Fourier transforms until only adequately sampled frequencies are transmitted. This amounts to broadening and sloping the sides of the response profiles and will lead to a good deal of overlap. For a given spacing of filters, it is a straightforward matter to calculate what the profiles should look like. It is not generally so straightforward to manufacture filters with such profiles.

## VI. CONCLUSIONS

Photometric measurements of satellites provide a potential wealth of information which can be used in the GEODSS program. Alas, errors arising in both the collection and processing steps often make the interpretation of the data difficult or ambiguous. These problems will now be examined in the context of reasonable expectations for the GEODSS program. There are four results which it should be possible to obtain from analyses of GEODSS data:

- any rotation period should be accurately measureable.
- a well corrected and reproducible magnitude should be derivable.
- a set of well corrected and reproducible color indices should be derivable.
- it should be possible to transform GEODSS magnitudes to other common systems.

It is assumed that the needed analytical tools exist. Only the question of suitability of data will be addressed here.

Measurement of rotation periods is likely to be an extremely important task in the GEODSS program. The EWP system is eminently suitable for this task as the only error present will be the completely unavoidable shot noise. Calculations based on GEODSS telescope parameters indicate that the magnitude limit for reliable period measurement is satisfactory. It may prove possible



in the future to draw additional conclusions as to the nature of satellites from analysis of the characteristic signature. For now it should be noted that the 500 Hz bandwidth of EWP is needed primarily to maximize the signal-to-noise ratio in the specular flashes used to measure period. Any other signature analysis is unlikely to need a bandwidth anywhere near as large.

A reliable magnitude would be used for size estimates and for measurement of the phase function. The G2-filter magnitude corrected for atmospheric extinction by means of a model incorporating C1 as an estimate of the slope of the spectral distribution should be more than adequate for this purpose. As a second choice the modified EWP magnitude might prove marginally acceptable. Atmospheric extinction varies only  $\pm 0.03^m$  across the width of the G2-filter, making this magnitude quite insensitive to the color. Thus, even though C1 is subject to aliasing, the error in the correction for atmospheric extinction should remain negligible. The drawback is the reduced throughput leading to increased shot noise error. In most applications, however, it should be possible to lengthen the integration time sufficiently to make up for the throughput loss.

The three color indices of MBP should prove useful as the basis for a satellite reflectivity classification. As was shown in section IV, these indices are very sensitive to satellite material,

taking on widely different values for various typical materials. It might be worthwhile attempting to "calibrate" the indices in terms of actual material samples. Even without this, subdivision of C0-C1-C2 space looks promising as a classification scheme. The color indices are subject to the same sort of error in the extinction correction as are the magnitudes. Errors in color indices, however, are typically considerably smaller than those in the magnitudes. The errors in the magnitudes are already small enough so it seems safe to assume that extinction correction errors will not be a problem for the color indices. Aliasing, on the other hand, will be a problem since it affects the desired quantity directly rather than at one remove, as in the case of the G2-filter magnitude. Removal of the aliasing by broadening the bandpasses would not be helpful, because this also eliminates the high frequency data such as the reflectivity spike at about  $.39\mu$  which characterizes solar panels. The aliasing of this important data gives rise to the problem of ambiguous classification, so the solution to the aliasing problem must be by means of additional bandpasses with wide Fourier transforms. The need for additional information may, however be met by sources other than additional photometric indices. The information in orbital characteristics, radar signatures, or other forms of data may be able to resolve the ambiguities. This would not solve the aliasing *per se* but it would allow unambiguous satellite classification, which is surely the important point.



The desire to transform MBP data to the UBV system is the least reasonable of the four expectations because there is no particular value in making the transformation. It is here that the problem of aliasing is most strongly felt. By measuring stars it will almost surely be possible to calibrate a transformation between MBP and UBV which works quite well for a large range of stars. This success would be misleading, however. Stellar spectra are almost completely devoid of strong features on the order of a few hundred angstroms wide, and so aliasing is almost no problem (until one looks at the  $0^m.003$  level) for stellar color indices. It is the presence of such features, however, which makes MBP of satellites interesting, and it is the data of these features which is subject to aliasing in both MBP and UBV photometry. Transformations between systems which contain aliased data can be expected to be non-linear and multi-valued - i.e., thoroughly intractable. In section V it was shown that, depending upon the application, the composite X3 could be "correctly" characterized by two values of C0 differing by nearly  $0^m.3$ . There is no reason to expect a transformation between MBP and UBV to be any better behaved than this.

The MBP system was conceived subject to the constraints of maximum utilization of incident photons and simplest possible instrument design and observing procedure. If these constraints

are lifted, then a system with more bandpasses and better profile shapes can be implemented, thus eliminating problems with aliasing. The cost from the standpoint of the GEODSS mission is a modest brightening of the limiting magnitude and a rather large increase in time devoted to photometric measurements. It appears difficult to justify this cost to meet the goal of satellite classification. The justification for these operational costs thus rests upon the need for an accurate transformation between MBP and UBV photometry. In the absence of any clear need for such a transformation, it is concluded that the needs of the GEODSS program would be well served by a system similar to the modified EWP supplemented on occasion by MBP.



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